Int. J. Solids Structures, 1976, Vol. 12, pp. 803–808. Pergamon Press. Printed in Great Britain

STRESS PULSES EMITTED DURING FRACTURE IN TENSION

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(Received 20 October 1975; revised 15 June 1976)

Abstract—This paper is concerned with the stress pulses which are emitted during the tensile fracture of glass rods of rectangular cross-section. Both a longitudinal and a flexural pulse were observed. The experimentally recorded pulse shapes are found to be in close agreement with those predicted by using a theoretical model proposed by Phillips and Kolsky. It is shown that the deviations noted in the earlier work with circular glass rods are due, primarily, to the effect of crack bifurcation and not to the geometry of the cross-section. Further, the phenomenon of crack bifunction is found to depend upon the magnitude of the applied stress required to initiate fracture, and to be essentially independent of the section geometry.

INTRODUCTION

The first detailed analytical consideration of the stress pulses emitted during the tensile fracture of brittle beam specimens appears to be due to Miklowitz [1]. The stress pulses emitted during the tensile fracture of glass rods of circular cross-sections have been discussed by Phillips [2] and Kolsky [3]. In these papers it is shown that the agreement between the theoretically predicted and the experimentally observed longitudinal stress pulses is very satisfactory, but that the agreement for the *flexural pulses* is not so close. It has been conjectured that these observed deviations may result from the fact that in the experiments which were compared with the theoretical predictions, extensive crack bifurcation had always taken place, whereas the theoretical predictions were based on the assumption that the fracture took place in a single plane. In this investigation it was found that this bifurcation is associated with the rather high tensile stresses which have to be applied to circular glass rods with shallow surface grooves before brittle fracture is initiated. With rods of square cross-section which were cut from glass plates, very much smaller longitudinal stresses were needed to initiate brittle fracture, and the fracture then traverses the specimen in a single plane. For a single plane fracture it was found that the shapes of not only the emitted longitudinal pulse but also of the emitted flexural pulse were in very good agreement with the theoretical predictions.

THEORETICAL PROCEDURE

The theoretical procedure used to compute the shapes of the fracture induced stress pulses is essentially similar to the one described in [2, 3].

Consider a square rod containing a corner flaw and subjected to a quasi-statically applied tensile stress. The crack begins to propagate when the applied stress infinitesimally exceeds the "Griffith stress" for the corner flaw, σ_0. As a result of the crack propagation across the loaded section, unloading waves are propagated in both axial directions. The intent of this section is to derive the expressions for these unloading waves.

The following simplifying assumptions are now introduced:

(1) The crack initiates at a corner of the rod section and propagates at a constant velocity V_c.

(2) Some distance away from the fracture plane only two types of wave are propagated, namely, a symmetrical longitudinal wave and an anti-symmetrical flexural wave. The work of Phillips [2] and Kolsky [3] has shown that the assumption that the generating stress for both the longitudinal and the flexural pulses may be considered as produced by the unloading stress over the fractured part of the specimen whereas the unfractured portion retains the stress it had prior to the commencement of fracture, leads to satisfactory agreement with the shapes of the longitudinal pulses observed some distance away from the fracture plane.

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(3) Finally, only the fundamental mode of both the longitudinal and the flexural wave propagation are assumed to be important.

The stress acting over the unfractured portion at $x = 0$ (see Fig. 1) may be lumped into two loading functions: the normal loading function $F(t) = \sigma_0 A_0 - \int_{A(t)} \sigma_0 \, dA$ and the flexural loading function $M(t) = -\int_{A(t)} \sigma_0 y \, dA$, where A_0 is the cross-sectional area of the beam, $A(t)$ is the area of the fractured surface, and t is time from fracture initiation. These stress resultants are assumed to act over the entire cross-section at $x = 0$. We now define a dimensionless longitudinal stress, $\Sigma_L = F(t)/4a^2\sigma_0$, a dimensionless flexural stress, $\Sigma_F = 3M(t)/4a^3\sigma_0$, and dimensionless time, $\tau = V_c t/2a$. A straightforward calculation yields the following expressions for Σ_L and Σ_F [4].

$$\Sigma_L(0,\tau) = \begin{cases} 1 & \tau < 0 \\ 1 - \frac{1}{4}\pi\tau^2 & 0 \le \tau \le 1 \\ 1 - \frac{1}{4}\pi\tau^2 + \tau^2 \cos^{-1}\left(\frac{1}{\tau}\right) - (\tau^2 - 1)^{1/2} & 1 < \tau \le (2)^{1/2} \\ 0 & \tau > (2)^{1/2} \end{cases} \quad (1)$$

and

$$\Sigma_F(0,\tau) = \begin{cases} 0 & \tau < 0 \\ -\frac{3\pi\tau^2}{4}\left(1 - \frac{8}{3\pi}\tau\right) & 0 \le \tau \le 1 \\ -\frac{3}{2}\left(\frac{4}{3}(\tau^2-1)^{3/2} + 2(\tau^2-1)^{1/2} \right. & \\ \quad \left. + \left(\frac{2}{3} - 2\tau^2\right) + \tau^2 \sin^{-1}\left(\frac{2}{\tau^2} - 1\right)\right) & 1 < \tau \le (2)^{1/2} \\ 0 & \tau > (2)^{1/2} \end{cases} \quad (2)$$

These "equivalent" stress pulses at $x = 0$ are shown in Fig. 1. For the "smooth" longitudinal pulse the ratio Λ/a was found to be about 10, where Λ is the pulse length. The geometrical dispersion of the longitudinal pulse may, therefore, be neglected [5] and hence $\Sigma_L(x, t) = \Sigma_L(0, t - x/c_0)$, where $c_0 = (E/\rho)^{1/2}$ is the extensional wave velocity, E and ρ are the Young's modulus and density respectively. The flexural pulse, on the other hand, is inherently dispersive and Timoshenko treatment [6] was used to compute the effect of the dispersion. The details of the

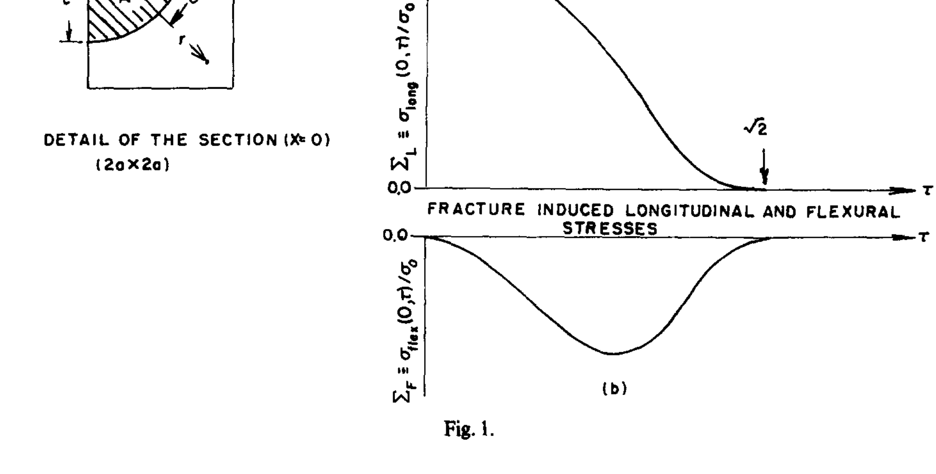

Fig. 1.

numerical computation may be found in [4]. Some choice has to be made regarding the magnitude of crack velocity V_c. It is assumed that $V_c = 0.38c_0$ which gives a total fracture time $T_f = 9\,\mu$sec. Computations were carried out for two values of T_f, 8 and 10 μsec and it was found that the numerical results are rather insensitive to such small variations in T_f. The numerical results at $x = 8$ in. for $T_f = 10\,\mu$sec are shown in Fig. 2.

EXPERIMENTAL PROCEDURE

Square beam specimens (1/2 × 1/2 in.) were cut from commerically available soda-lime glass plates of 1/2 in. thickness. A typical specimen is shown in Fig. 3. In order to pre-determine the plane of fracture, a shallow corner notch was introduced at one of the edges of the specimen. Four strain gages (Baldwin-Hamilton-Lima, SR-4, Type C-10, 1000Ω) were cemented to the specimen at two gage stations located at suitable distances from the fracture plane. In some of the early experiments fracture was found to take place near one of the grips. In order to make this less likely the compliance of the grips was increased by inserting thin rubber sheets between the specimen and the steel plates, and by the use of a flexible epoxy cement (TRA-BOND BB-2133) to form the glass-rubber and the rubber-steel bonds.

Now the primary purpose of this investigation was to study the emitted flexural pulses. If the beam specimen sustains any bending moment prior to fracture initiation, the fracture would result in extraneous flexural pulses. In order to minimize the pre-fracture bending moment, steel wire ropes of 1/8 in. dia. were used to connect the grips to the loading fixtures of an Instron tension testing maching. A technique similar to the one described in [2] was used to trigger the oscilloscopes (Tektronix type 556).

RESULTS AND DISCUSSION

In all the experiments carried out with square glass rods (1/2 × 1/2 in. nom.) the crack propagation was found to be contained in a single plane perpendicular to the axis of the rod. This is in marked contrast to the similar work on circular glass rods (1/2 in. nom.) where extensive crack bifurcation was observed. The latter phenomenon has been reported earlier in [2, 3]. In order to determine the influence of section geometry on the crack propagation behavior, a series

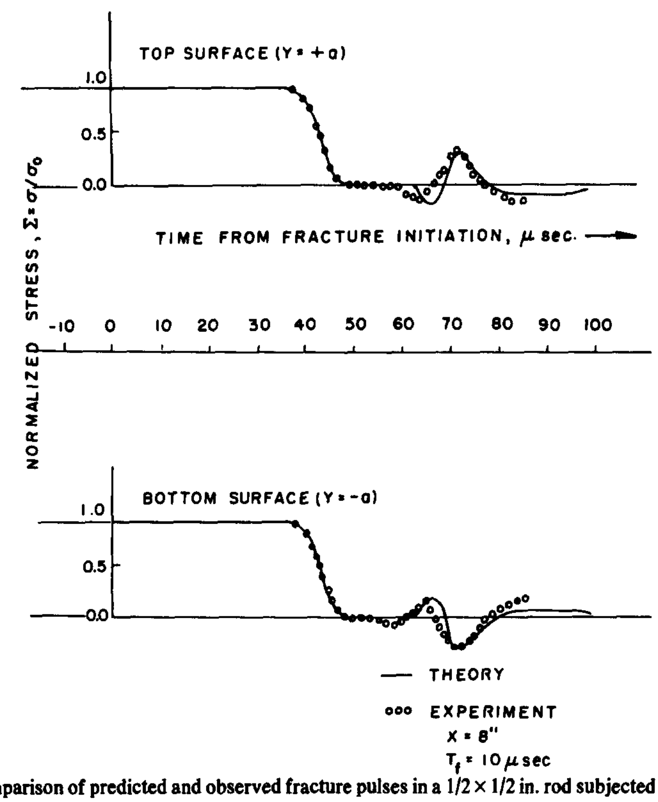

Fig. 2. Comparison of predicted and observed fracture pulses in a 1/2 × 1/2 in. rod subjected to tension.

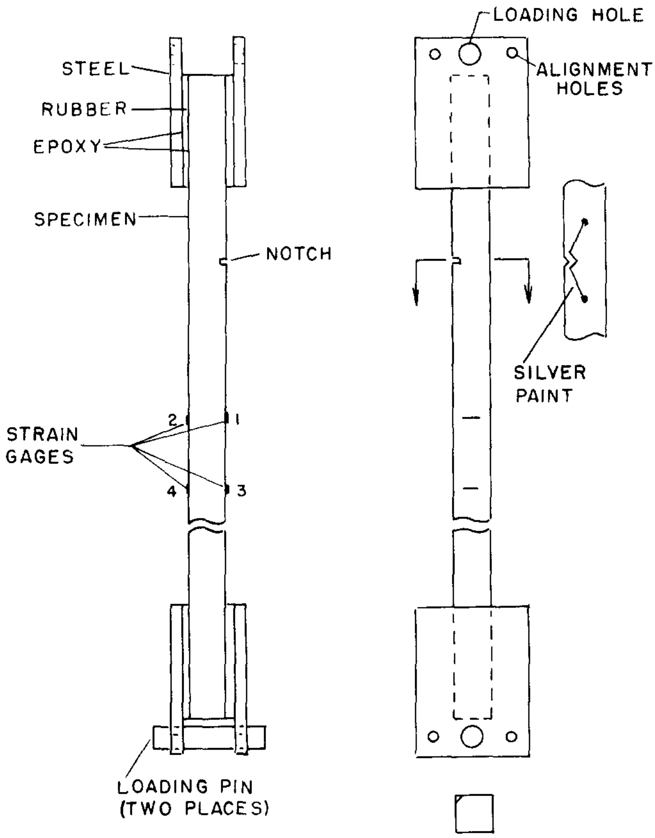

Fig. 3. Detail of a typical tensile specimen.

of fracture tests was conducted on circular and square glass rods, in pure tensile and pure flexural loading, and the results are discussed next.

The fracture strength, σ_0, (defined as the nominal tensile stress required to initiate fracture) of the plate glass used in these experiments in its as-received (i.e. un-notched) condition was found to be about 6000 psi for a one sec loading time, which was also the fracture strength quoted by the manufacturer [8]. The fracture strength of the *notched* square rods was found to be considerably lower than 6000 psi depending upon the "severity" of the notch introduced which, in turn, depends upon the notch depth and the notch tip radius. Throughout this investigation, the notch tip radius was held constant by using the same diamond wheel to introduce the notches. Thus the "severity" of the notches was controlled by controlling the depth of the notch. The static ultimate tensile stress for *circular* glass rods was quoted in [2] to be about 16,000 psi, whereas the observed fracture strength for *scratched* rods was reported to be 5350 psi ± 10%. The results of the present investigation showed that with essentially similar notches, the fracture strength of the circular rods was about *twice* that of the square rods. Some possible reasons for this observed difference in the fracture strength are discussed in the Appendix. When circular rods were weakened by introducing notches of increasing depth, the extent of bifurcation was found to decrease, as also did the fracture stress, σ_0. With a sufficiently deep notch, the crack was found to propagate in a single plane (Fig. 4). Conversely, when square rods, with extremely shallow notches, were subjected to fracture in simple tension, the initial mirror-like surface was followed by hackle structure which indicated a tendency to bifurcation [7]. Similar results were obtained when rods of square (1/2 × 1/2 in. nom.) and circular (1/2 in. dia.) cross-section were fractured in pure flexure. It is concluded that the phenomenon of crack bifurcation is essentially independent of the section geometry. Furthermore, the marked difference in crack bifurcation behavior between circular and square rods of comparable section dimensions is due, primarily, to the large difference in the tensile stress required to initiate fracture.

Returning to the main objective of this work, the experimentally observed and the theoretically postulated shapes of the stress pulses emitted during the tensile fracture of square rods are

Fig. 4. Two views of fracture surfaces produced by tensile fracture of 1/2 in. circular glass rods. Notch depth decreases from left to right.

compared in Fig. 2 where $\Sigma = \Sigma_L + \Sigma_F$. The first pulse to arrive is the unloading pulse of compression. This is followed by a short quiescent period. The antisymmetric flexural pulse arrives next. The agreement is considered to be extremely good for both the longitudinal and the flexural pulse. It is concluded that the discrepancy between the theoretical and the experimental shapes of the *flexural* pulse reported by Phillips [2] and Kolsky [3] appears to be due, primarily, to the phenomenon of extensive crack bifurcation which accompanied the experiments reported in these references.

CONCLUSIONS

Stress pulses emitted during the tensile fracture of square glass rods have been studied. The fracture was found to propagate in a single plane. The shapes of both the emitted longitudinal pulse and of the emitted flexural pulse were found to be in very good agreement with the theoretical predictions. It is concluded that the discrepancy between the shapes of the experimentally observed and the theoretically predicted flexural pulses which was reported in [2, 3] is due, primarily, to the phenomenon of extensive crack bifurcation which occurred in the experiments reported in the earlier work. Furthermore, it has been shown that the phenomenon of crack bifurcation is essentially independent of the section geometry.

Acknowledgements—The author wishes to express his appreciation to Prof. H. Kolsky for suggesting the problem and for his valuable advice in the course of this investigation. Thanks are due to Mr. R. Stanton for technical assistance, to Mrs. E. Fonseca for her help in the preparation of the manuscript, and to Miss E. Addison for her assistance with the illustrations. The financial support of the author by the National Science Foundation under grant ENG 73-08060 is gratefully acknowledged.

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APPENDIX

(1) ***Surface damage due to manufacturing processes***: The circular rods are manufactured by an extrusion process and hence suffer relatively little surface damage. The plate glass (or float glass) is manufactured by a float process and is subsequently hand cut, thus suffering considerable surface damage, particularly at the edges. However, in all the experiments conducted with *notched* specimens, the fracture was found to propagate at the notch, thereby indicating that the stress concentration due to the notch was more severe than that due to any of the pre-existing flaws. Therefore, in these experiments surface damage was not a parameter governing the fracture strength. (2) *Residual stresses*: In unannealed glasses there are compressive residual stresses near the surfaces which tend to increase the observed fracture strength. The circular rods are not annealed to the same extent as the plate glass. (3) *Composition*: The circular rods are manufactured from borosilicate glass whereas the plate glass was made from soda-lime glass. This *may* contribute to the observed differences in the fracture strength, although the manufacturer did not specify the effect of composition on the static ultimate tensile stress [9].